Lagrangian Analysis and Forecasting in the Oceans and Coastal Zones

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LONG-TERM GOALS

The goal of this research program is to develop a more detailed and systematic understanding of transport and mixing in the oceans and coastal zones. Ultimately, the program will lead to Lagrangian forecasting, that is, the ability to make specific deterministic predictions about the advection and diffusion of passive scalars in the ocean. To accomplish this, it will be necessary to unify several rapidly advancing areas: aspects of dynamical systems theory developed to identify transport barriers and coherent Lagrangian structures, Lagrangian stochastic models of turbulent diffusion, and Eulerian observation data, e.g. sea surface height (SSH) from the TOPEX/Poseidon altimeter for global/regional studies and high frequency (HF) radar for coastal studies.

OBJECTIVES

We endeavor to advance dynamical systems templates and the methodology for applying them to recent observation data of the ocean. In particular, we test a new dynamical system theory based on finite-time Lyapunov exponents for identifying Lagrangian barriers in HF radar data of Monterey Bay which dictate where passive particles, such as contaminants released from Moss Landing, will be advected. We address the specific question of whether a given parcel will remain in the bay indefinitely, or will be advected efficiently from the bay.

APPROACH

The study of transport and mixing in fluids problems has long been approached from a Eulerian perspective, i.e. observing quantities such as velocity and concentration of scalar material, such as a contaminant in a bay, as it moves past a stationary reference frame. The evolution of velocity and concentration in a Eulerian reference frame are described by the advection-diffusion equation. The problem with this equation is that it contains nonlinear advection terms and so when applied to a turbulent flow, the evolution equations for the mean velocity and mean concentration are not closed. In other words, they involve higher order statistics such as the Reynolds stresses and scalar density

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14. ABSTRACT
The goal of this research program is to develop a more detailed and systematic understanding of transport and mixing in the oceans and coastal zones. Ultimately, the program will lead to Lagrangian forecasting, that is, the ability to make specific deterministic predictions about the advection and diffusion of passive scalars in the ocean. To accomplish this, it will be necessary to unify several rapidly advancing areas: aspects of dynamical systems theory developed to identify transport barriers and coherent Lagrangian structures, Lagrangian stochastic models of turbulent diffusion, and Eulerian observation data, e.g. sea surface height (SSH) from the TOPEX/Poseidon altimeter for global/regional studies and high frequency (HF) radar for coastal studies.

15. SUBJECT TERMS

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fluxes. Closure approximations introduced to overcome this problem depend on the concentration field itself, and so are not uniformly valid. The other alternative is to observe the flow from a Lagrangian perspective, tagging individual particles and noting the property changes that each particle experiences. A Lagrangian perspective allows the flow and concentration to be described in a reference frame that follows a point moving with the fluid. The primary advantage of using a Lagrangian framework is that the time derivative following the motion includes the nonlinear advection terms implicitly. The analog of the advection-diffusion equation in a Lagrangian framework is thus trivial, stating that a fluid particle retains its original concentration as it moves through the fluid. Since a particle conserves it concentration, changes in the concentration field occur solely due to redistribution throughout the fluid. The closure problem still remains, but approximations now only involve the velocity field and not the concentration field. In addition to this theoretical advantage, a Lagrangian approach, i.e. a particle tracking approach does not require the solution of an additional partial differential equation and is thus computationally more efficient.

During the first year of this program, the key individuals in this study include Francois Lekien at Caltech, George Haller at Brown University, Andy Reynolds at the Silsoe Research Institute, the PI and co-PI, Chad Coulliette and Jerrold Marsden, respectively.

WORK COMPLETED

We developed and tested a new approach for identifying Lagrangian coherent structures on recent HF radar current measurement data from Monterey Bay. This new approach is completely different from previous efforts in that we are able to identify all Lagrangian coherent structures simultaneously and we successfully tested it as a predictive tool. This statement requires some clarification: all previous methods which used dynamical systems theory to identify Lagrangian coherent structures and predict the behavior of qualitatively different regions of the flow required 1) a heavy filtering of the measured HF radar data and 2) a Eulerian prediction of the heavily filtered data in order to predict behavior beyond the available data. Our new approach described in this report successfully circumvents both of these limitations. We have encoded this new approach into a software package called MANGEN, which includes a graphical user interface. The user interface is presently compatible with Windows platforms only, but soon will be ported to a wide variety of UNIX/Linux platforms.

RESULTS

We demonstrate in this report that a combination of accurate current measurements and recent developments in the mathematics of nonlinear systems has the potential to reduce the damaging effects of coastal pollution. The focus of our study is the Elkhorn Slough, located near Moss Landing harbor of Monterey Bay. The Elkhorn Slough is a regular source of organic contaminants such as dichlorodiphenyl-trichloroethane (DDTs) and polychlorinated biphenyl (PCBs) from agricultural run-off, phthalic acid esters (PAEs) from plasticizer manufacturing, insecticidal sprays, wetting agents and repellents, and polycyclic aromatic hydrocarbons (PAHs) from the combustion of natural fossil fuels[1-3].

We examine high-frequency (HF) radar measurements[4-8] of near-surface currents in Monterey Bay, and identify a set of fluid particles that governs the chaotic mixing of contaminants over finite intervals of data. Specifically, we find a highly convoluted Lagrangian coherent structure composed of a line of fluid particles, that repels nearby fluid parcels and hence acts as a barrier between two different kinds of motion: recirculation and escape from the bay. If pollution is released on the appropriate side of this

moving fluid structure, then contamination will quickly clear from coastal regions and head towards the open ocean. On the other hand, release of pollution on the other side of the repelling material line will result in sustained recirculation of the contaminant in the bay. As a result, pollution can be quickly cleared from the coast if one ties its release to a Lagrangian forecasting of Monterey Bay.
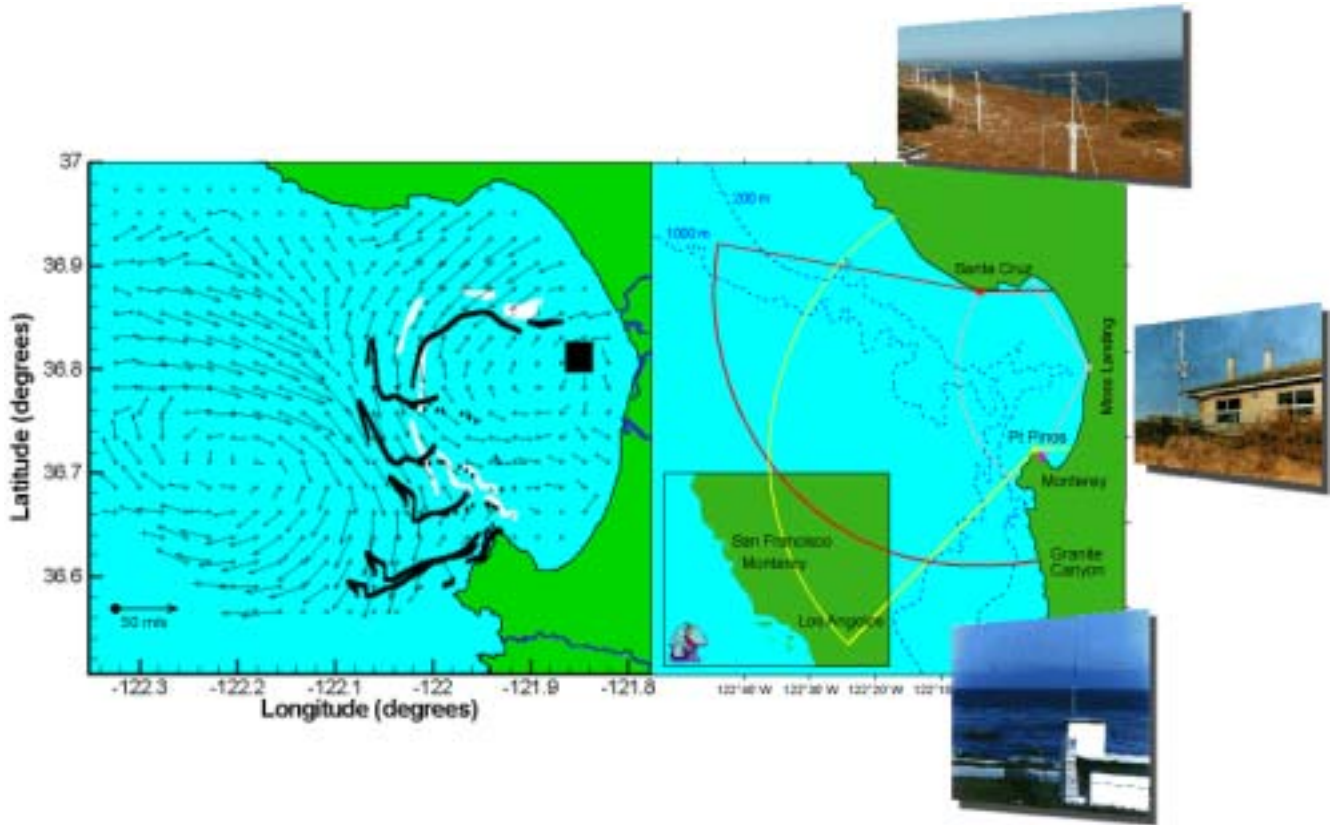

Figure 1. Instantaneous near-surface velocities shown as black arrows (left panel) at 08:00 GMT, August 8, 2000, obtained from the three HF antennas in Monterey Bay (right panel) at Santa Cruz, Point Pinos and Moss Landing [4-8]***. Shape and position of parcels of two parcels (left panel) of contaminants released from the same position near Moss Landing at 2200 GMT August 6, 2000 (white) and 0900 GMT August 7, 2000 (black), respectively. The parcels shown correspond to 2200 GMT 6 August 2000, 0800 GMT 7 August 2000, 0400 GMT 9 August 2000, 0500 GMT 10 August 2000, 0500 GMT 11 August 2000, and 1200 GMT 15 August 2000. Note that the white parcel remains in the bay and the black parcel departs from the bay.***

In contrast to earlier approaches to timed pollution release from holding tanks[9-16], we avoid the use of simplified models and target measured ocean data directly. This strategy accommodates constantly changing real-life flow conditions. Another novel feature of our study is the use of finite-time dynamical systems methods[17-19] for the analysis of HF radar data. The recent interest in the development and application of such methods stems from the realization that mixing in meso-scale geophysical flows is governed by coherent structures of finite lifespan[17-19]. The presence of finite-time coherent features in measured geophysical flow data prevents the application of the statistical theory of turbulent mixing[20,21], while the temporal irregularity and spatial complexity of such data renders the techniques of chaotic advection[22-26] inapplicable.

Our analysis makes use of high frequency (HF) radar technology[4-8], which is now able to resolve time-dependent Eulerian flow features in surface currents along coastlines. Such an HF radar installation has been operating in Monterey Bay since 1994[4,5]. In our study, we use recent data from August 2000, measured from three HF radar antennas with overlapping footprints in the bay (Fig. 1). We use continuous observational data, binned every hour on a horizontal uniform grid with 1 km by 1 km intervals. An example of an HF radar footprint from 05:00 GMT, August 12, 2000 is shown in the left panel of Fig. 1. Interpolating and integrating measured velocity slices, we obtain approximations of individual fluid trajectories, or even whole parcels of fluid. It then becomes possible to track and compare different evolutions of a fluid parcel, a model for a blob of contaminant, released at the same coastal location at different times. We show the results of two such experiments as a white parcel and a black parcel in the left panel of Fig. 1. Note that in one case the contaminant parcel remains in the bay, whereas in the other case it clears from the bay towards the open ocean. The latter scenario is highly desirable, because it minimizes the impact of the contaminant on coastal waters. This observation motivates us to understand and predict different evolution patterns of the same fluid parcel, depending on its initial location and time of release.

To understand the evolution of fluid parcels, we use a geometric description of fluid mixing from nonlinear dynamical systems theory. Two-dimensional time-periodic fluid flows have long been known to produce *chaotic advection*, i.e., irregular stirring of fluid parcels. Instrumental in this stirring are stable and unstable manifolds of distinguished periodic fluid trajectories. Stable (resp. unstable) manifolds are material curves formed by fluid trajectories that converge to (resp. diverge from) an underlying periodic trajectory. For near-incompressible flows, the convergence within a stable manifold causes the manifold itself to repel nearby fluid parcels. As a result, stable manifolds act as repelling material lines that send fluid blobs on their two sides to different spatial regions. For the same reason, unstable manifolds act as attracting material lines, targets along which fluid blobs spread out and form striations. We refer attracting and repelling material lines jointly as *hyperbolic material lines*. Recent progress in nonlinear dynamical systems has extended the above geometric picture to velocity fields with general time dependence. As it turns out, families of hyperbolic material lines continue to organize finite-time mixing even in turbulent flows. Several numerical algorithms and theoretical criteria have been proposed to identify hyperbolic material lines in numerical and experimental data sets. Here we use the Direct Lyapunov Exponent (DLE) algorithm, which starts with the computation of the flow map, the map that takes an initial fluid particle position $\mathbf{x}_0$ at time t_0 to their later position $\mathbf{x}(t,\mathbf{x}_0)$ at time t. One then takes the largest singular value $\sigma_t(\mathbf{x}_0)$ of the derivative of the flow map with respect to $\mathbf{x}_0$. More specifically, one calculates the scalar field $\sigma_t(\mathbf{x}_0)$ as the largest eigenvalue of the Cauchy-Green strain tensor $\left[\partial\mathbf{x}(t,\mathbf{x}_0)/\partial\mathbf{x}_0\right]^T\left[\partial\mathbf{x}(t,\mathbf{x}_0)/\partial\mathbf{x}_0\right]$, with the superscript T referring to the transpose of a matrix. As argued in [19], repelling material lines are local maximizing curves of $\sigma_t(\mathbf{x}_0)$, which will allow us to capture these material lines at time t_0 as ridges of the scalar field $\sigma_t(\mathbf{x}_0)$. The same procedure performed in backward time (i.e., for $t<t_0$) would render attracting material lines as ridges of $\sigma_t(\mathbf{x}_0)$.

Fig. 2 shows several different distributions of DLE(t,x_0)--defined as logarithm of the scalar field $\sigma_t(\mathrm{x}_0)$—over the initial grid. In agreement with the above general theory, local maximizing curves or ridges on this plot form repelling material lines, i.e., barriers. Note the convoluted maximizing curve that attaches to the southern coastline of the bay. Because of the difference in time, the location of release happens to fall on different sides of this main repelling material line, resulting in qualitatively different behavior for the two parcels. The white parcel, released on August 6, quickly clears from the

bay, while the black parcel, released a few hours later, spreads and recirculates in the bay. The former behavior is clearly highly desirable for parcels of pollutants.
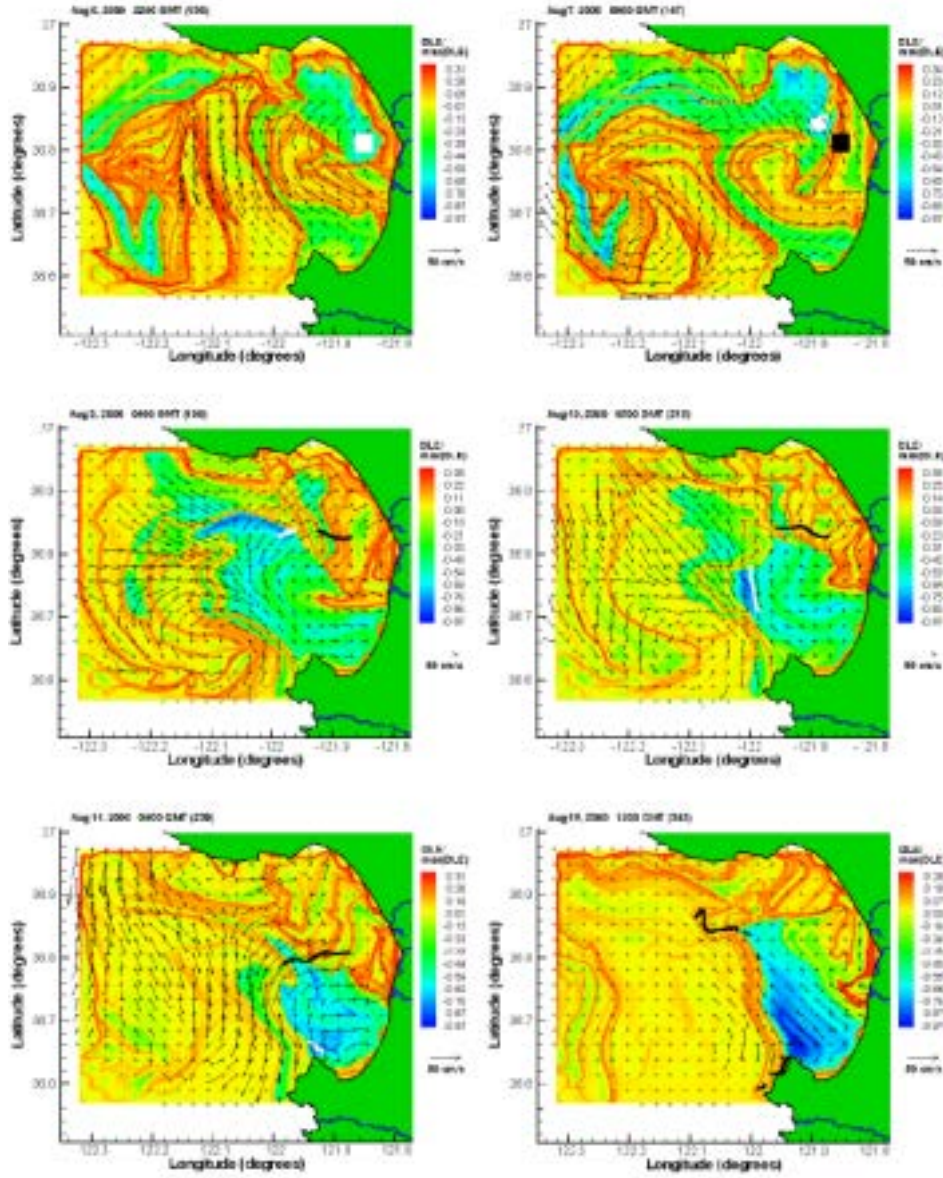

Figure 2   Lagrangian coherent structure of Monterey Bay at 0600 GMT on August 8, 2000, depicted using Finite-time Lyapunov exponent computed directly from particles advected with HF radar measured currents. The vectors show the Eulerian structure of the HF measured currents. Two parcels (black and white) of contaminants released from the same position near Moss Landing at 2200 GMT August 6, 2000 and 0900 GMT August 7, 2000, respectively.

We imagine now that a pipeline carries the contaminants from the Moss Landing area and releases them in the bay. The inset in Fig. 3 shows this setting as well as the intersection of a particular Lagrangian coherent structure (LCS) with the axis of the pipeline— defined as a local maximum of the DLE of the flow. We know from our previous analysis that contaminants released on the right of this particular LCS will be advected immediately into the open ocean to be safely dispersed, but those released on the left of this particular LCS pollute the bay for an indefinite period. The red curve in Fig. 3 shows the time evolution of the LCS intersection with the prolongation of the pipe. The black and white squares in the left panel Fig. 3 denote the release time of the two parcels considered in the

previous section. Note that the release time of the white and black parcel is separated by only a small interval. The release times and location of these parcels were chosen specifically to highlight the chaotic behavior of mixing in Monterey Bay. The parcels are released from exactly the same location, only separate by a few hours, but yet their ultimate fates are completely opposite. Plotting the intersection of this LCS and the axis of the pipeline gives us a convenient procedure to predict the behavior of the parcels. We can see from this chart that the optimal release time to insure that the contaminants are carried out to sea would be when the red curve is the furthest below (closer to the shoreline) the pipeline location (that is, the end of the pipe where contamination is released).

Fig. 3 shows the spectrum corresponding to the time evolution of the DLE maximum defining the particular LCS that determines if parcels will exit or remain in the bay. Note that there are seven peaks in this spectrum. The 1st and 5th peak are worthy of particular discussion: the 5th peak has a wavelength of approximately 24 hours, which corresponds to the diurnal or tidal constituent. This is a good check because the tidal influence should be noticeable. Many environmentalists argue that we should simply release contaminants at high tide. But, we can see that the highest peak however corresponds to a wavelength of around 240 hours, or approximately 10.3 days. Thus the primary factor determining if contaminants will remain in the bay or be swept out to sea is not tidal. Simply releasing at high tide in Monterey Bay will not be close to optimal. The only way to determine the optimal release time is to do an analysis similar to that described here.

Before we can use this data for a true prediction, we must compute DLE contours using only information up to the "present time". Let us consider the white particle release time, 2200 GMT Aug 6, 2000, as the present time. The result of this computation is shown in Fig. 3. The red curve in the right panel represents the original DLE computation using a 200 hour advection time, and the green curve represents this new DLE "present time" computation. The results are quite amazing. In the range of 135 hours before the "present time" to 55 hours before, the maximum error is less than 0.01 degrees, which is a 5% error, if the reference length scale is based on the width of the bay. Stating this another way: using only 15 hours to compute DLE contours, it is possible to make a prediction about the location of the Lagrangian barrier which will determine if contaminants released from Moss Landing will stay in the bay or exit immediately from the bay.

We can further increase the accuracy and longevity of our prediction by constructing an approximation to the curve in Fig. 3 from the seven dominant frequencies of Fig. 3. The amplitude of each component can be chosen based on the amplitude (or area) of each peak. The result is shown as the blue curve in Fig. 3, which does an amazing job of capturing the large scale oscillations of the DLE ridge and thus could be used to predict good release times. For example, using only information from the simulated "present time" DLE computation shown by the green curve, we can predict (blue curve) that the first release time that will efficiently advect contaminants from Moss Landing out to sea will be between 0600 GMT August 6, 2000 (hour 143 in Fig 3) and 1800 GMT Aug 12, 2000 (hour 299 in Fig. 3). This corresponds closely with the correct release time computed from all available current measurements (red curve), which gives a release period of 0800 August 6, 2000 (hour 145 in Fig. 3) to 2100 August 10, 2000 (hour 254 in Fig. 3). A second release period is also predicted with excellent accuracy. The predicted release period is 2300 GMT August 15, 2000 (hour 376 in Fig. 3) to 2100 GMT August 22, 2000 (hour 542 in Fig. 3), compared to an actual release period of 0500 GMT August 17, 2000 (hour 406 in Fig. 3) to 2200 GMT August 22, 2000 (hour 543 in Fig. 3). It is remarkable that such little data prior to the release of the white parcel can successfully predict a release period more than two weeks into the future from only five days of HF radar data.
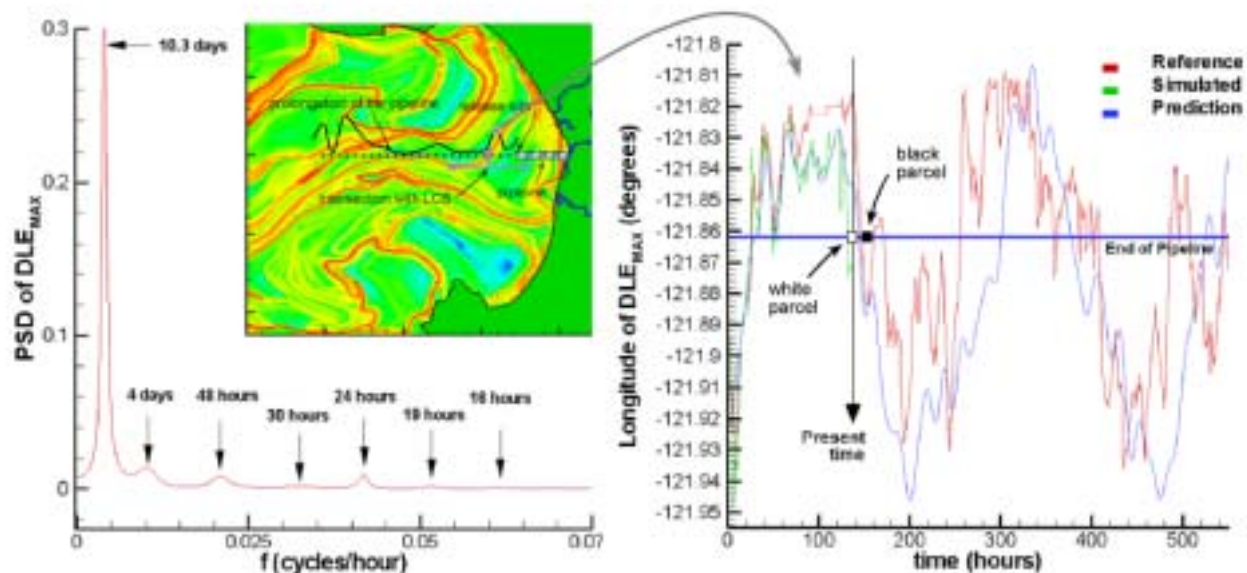
Figure 3 Oscillation of the DLE ridge along an axis that intersects the pipeline (the time is referenced to 0700 GMT August 1, 2000). The horizontal line corresponds to the open end of our imaginary pipeline that releases contaminants. The white square represents the release time and release longitude of the white parcel and the black square represents the release time and release longitude of the black parcel.   A spectrum of the time series of DLE ridge oscillations (red curve) is shown in the left panel.   There are seven peaks, with the majority of the energy contained in oscillations with a 10.3 day period. There is also significant energy at 24 hours, 48 hours and 4 days. The green curve is the "Simulated" or present time curve computed only with information up to the release time of the white parcel.   The blue curve is the "Predicted" value of the ridge oscillation.

IMPACT/APPLICATIONS

There is a tremendous potential impact for this study in any area where it is desirable to predict where any passive tracer will drift. We have specifically shown by example how this newly developed approach could be used to predict if contaminants released in a bay will be advected quickly from the bay or remain in the bay indefinitely. Although there are many laws in our country that specify how contamination must be treated chemically before being released into our coastal zone, there are not any laws (to our knowledge) specifying precisely when contaminants can be released. This is because up until now it has been thought that predicting where contaminants in a bay, estuary or other coastal zones of complex geometry will be advected is impossible. Perhaps lawmakers can use the information about this new approach described in this report to regulate more precisely when contaminants can be released in our coastal zone. In addition, there are many other possible applications of this approach. We could use a similar approach to predict where oil spills will be advected, to determine more exactly how currents alter the trajectory of drogues or underwater gliders, or possibly to reduce the search area for persons that have fallen overboard.

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TRANSITIONS

Several research groups at universities throughout the world have requested copies of the beta release of our MANGEN software at the most recent General Assembly of the European Geophysical Society or independently and are presently using it in their programs.

RELATED PROJECTS

Recent progress in nonlinear dynamical systems has revealed the fundamental role of Lagrangian coherent structures in fluid transport. While several algorithms exist for the extraction of such structures from numerical flow models, the relationship between model Lagrangian structures and their counterparts in the true flow (if any) has remained unclear. Recent work by George Haller (Brown University) has revealed that Lagrangian coherent structures found in model data tend to give accurate predictions for similar flow structures in the real flow. This work was in part inspired by this program at Caltech to interpret Lagrangian predictions obtained from HF radar data. An ongoing effort is to use Haller's results in deriving error bars on Lagrangian predictions for the Monterey Bay. http://www.cfm.brown.edu/people/haller

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Coulliette, C. and Wiggins, S., "Intergyre transport in a wind-driven, quasigeostrophic double gyre: An Application of Lobe Dynamics" (black & white printing) *Nonlinear Processes in Geophysics*, **7**(1), 59--86 (2000).

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PATENTS

MANGEN software (patent pending).